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# DYNAMICS OF A VAPOR SHELL AROUND A HEATED PARTICLE IN A LIQUID

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*The dynamics and heat and mass exchange of a vapor bubble containing a heated particle is studied in relation to the problem of vapor explosions. It is shown that the process involves two stages: dynamic stage and thermal stage. The dynamic stage is characterized by pressure fluctuations and a rapid increase in the thickness of the vapor layer. It is shown that the simplifying assumptions of the constancy of assumptions of constant heat conductivity of the vapor and linear temperature profile in the vapor layer lead to qualitatively incorrect results.*

**Key words:** *vapor explosion, boiling, vapor bubble.*

The problem of the dynamics of a single vapor bubble containing a heated particle is considered in a spherically symmetric formulation in relation to the problem of vapor explosions [1, 2].

A practical application of the problem considered is the analysis of the possible power plant accidents resulting in the entry of hot nuclear fuel particles into cold water in the nuclear reactor cooling system. This leads to the explosive boiling of the liquid around the particles and a sharp pressure rise in the system. An attempt to determine the magnitude of the acoustic impulse in the case of film boiling on spherical particles was undertaken in [3]. That paper, however, contains a number of inaccuracies. The dynamics of the radial motion of the vapor shell around a heated particle subjected to a shock wave was studied numerically in [4]. The effects exerted on the shell by the shock-wave pressure gradient and the ratio of the initial thickness of the vapor shell to the particle radius. This was done using a generalization of the model of vapor bubble dynamics proposed in [5]. A review of papers on bubble dynamics is given in [6, 7].

**Formulation of the Problem. Basic Equations.** A hot particle or drop (molten metal, etc.) enters a cold liquid (water). It is assumed that the particle temperature far exceeds the boiling temperature of the liquid. We will investigate this problem using a spherically symmetrical model. In the given formulation, the equations of continuity, heat influx, and state for the vapor are written as [5]

$$\frac{\partial \rho_g^0}{\partial t} + \frac{1}{r^2}\,\frac{\partial w_g \rho_g^0 r^2}{\partial r} = 0,$$

$$\rho_g^0 c_g\Big(\frac{\partial T_g}{\partial t} + w_g\,\frac{\partial T_g}{\partial r}\Big) = \frac{1}{r^2}\,\frac{\partial}{\partial r}\Big(\lambda_g r^2\,\frac{\partial T_g}{\partial r}\Big) + \frac{dp_g}{dt},$$

$$p_g = \rho_g^0 R_g T_g, \qquad d \leqslant r \leqslant a(t),$$

where $c_g$, $\lambda_g$, $R_g$, and $w_g$ are the specific heat at constant pressure, the heat conductivity, the gas constant, and the vapor velocity, $d = \text{const}$ is the particle radius, $a(t)$ is the outer radius of the vapor layer, $\rho$ is the density, $\rho^0$ is the true density, $p$ is the pressure, $T$ is the temperature, $r$ is the radial Euler coordinate, and $t$ is time.

It is assumed that in contrast to the vapor temperature and density, the vapor pressure is uniform, which is valid for a wide class of problems in which the gas velocity is much lower than the sound velocity in the gas [5, 8].

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For the particle and the surrounding liquid, the equations of continuity, heat influx, and state become [8, 9]

$$\rho_d^0 c_d \frac{\partial T_d}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\Big(\lambda_d r^2 \frac{\partial T_d}{\partial r}\Big),$$

$$\rho_d^0 = \text{const}, \qquad w_d = 0, \qquad 0 \leqslant r \leqslant d,$$

$$\rho_l^0 c_l \Big(\frac{\partial T_l}{\partial t} + w_l \frac{\partial T_l}{\partial r}\Big) = \frac{1}{r^2}\frac{\partial}{\partial r}\Big(\lambda_l r^2 \frac{\partial T_l}{\partial r}\Big),$$

$$\rho_l^0 = \text{const}, \qquad w_l r^2 = w_{la} a^2(t), \qquad a(t) \leqslant r,$$

where $w_{la} = w_l(a,t)$ is the liquid velocity at the interface with the vapor layer.

The boundary conditions for the given problem are written as follows [8]:

$$\begin{aligned}
r = 0&: \quad \frac{\partial T_d}{\partial r} = 0,\\
r = d&: \quad \lambda_g \frac{\partial T_g}{\partial r} = \lambda_d \frac{\partial T_d}{\partial r}, \quad w_{gd} = 0, \quad T_g = T_d,\\
r = a(t)&: \quad \lambda_l \frac{\partial T_l}{\partial r} - \lambda_g \frac{\partial T_g}{\partial r} = jL, \quad T_{ga} = T_{la} = T_{gs}(p_g),\\
&\quad\; j = \rho_l^0 (a' - w_{la}) = \rho_g^0 (a' - w_{ga}), \quad a' = \frac{da}{dt},\\
r \to \infty&: \quad T_l \to T_\infty = \text{const}, \quad p \to p_\infty = \text{const}.
\end{aligned} \tag{1}$$

Here $j$ is the phase-transition rate referred to unit surface area ($j > 0$ corresponds to vaporization) and $L$ is the specific heat of vaporization; the subscript $s$ refers to the parameters on the saturation line.

The pressure and temperature on the saturation line are related by the Clausius–Clapeyron equation

$$\frac{dT_{gs}}{dp} = \frac{T_{gs}(1 - \rho_{gs}^0/\rho_l^0)}{L\rho_{gs}^0}.$$

Bubble dynamics is described by the Rayleigh equation [6]

$$a\frac{dw_{la}}{dt} + \frac{3}{2} w_{la}^2 = \frac{p_g - 2\sigma/a - p_\infty}{\rho_l^0},$$

where $\sigma$ is the surface-tension coefficient.

In the problem considered, one needs to allow for the temperature dependence of the heat conductivity of the vapor, which is taken in the form

$$\lambda_g(T) = AT + B.$$

The remaining characteristics of the media ($\lambda_d$, $\rho_d^0$, $c_d$, $c_g$, $L$, $\lambda_l$, $\rho_l^0$, and $c_l$) can be considered constant.

Substituting the continuity equation for the vapor into the heat-influx equation and using the equation of state, we write the system of equations for the vapor phase as [8]

$$\frac{\partial T_g}{\partial t} + w_g \frac{\partial T_g}{\partial r} = \frac{1}{c_g \rho_g r^2}\frac{\partial}{\partial r}\Big(\lambda_g(T) r^2 \frac{\partial T_g}{\partial r}\Big) + \frac{1}{c_g\rho_g}\frac{dp_g}{dt},$$

$$\frac{dp_g}{dt} = \frac{3(\gamma - 1)}{a^3 - d^3}\lambda_g(T) r^2 \Big(\frac{\partial T_g}{\partial r}\Big)_{a-d} - \frac{3\gamma p_g}{a^3 - d^3} a^2 w_{ga},$$

$$w_g = \frac{\gamma - 1}{\gamma p_g r^2}\lambda_g(T) r^2 \Big(\frac{\partial T_g}{\partial r}\Big)_{r-d} - \frac{(r^3 - d^3)(\gamma - 1)}{3\gamma p_g r^2}\frac{dp_g}{dt},$$

$$d \leqslant r \leqslant a(t),$$

where the subscript $a{-}d$ refers to the operation $f_{a-d} = f(a) - f(d)$.

For $r = a(t)$, boundary conditions are imposed on the moving boundary. For convenience of numerical calculations, this boundary was "frozen" by the corresponding transformation of coordinates:

$$0 \leqslant r \leqslant d: \qquad \eta = r/d, \qquad 0 \leqslant \eta \leqslant 1,$$

$$d \leqslant r: \qquad \xi = (r-d)/(a-d), \qquad 0 \leqslant \xi,$$

$$\Big(\frac{\partial}{\partial t}\Big)_r = \Big(\frac{\partial}{\partial t}\Big)_\xi - \frac{a\xi}{a-d}\,\frac{\partial}{\partial \xi}, \qquad \frac{\partial}{\partial r} = \frac{1}{a-d}\,\frac{\partial}{\partial \xi}.$$

Thus,

$$r = 0: \quad \eta = 0, \qquad\qquad r = d: \quad \eta = 1, \quad \xi = 0,$$

$$r = a(t): \quad \xi = 1, \qquad\qquad r \to \infty: \quad \xi \to \infty.$$

The transformed system of equations and boundary conditions was studied by numerical methods.

**Results of Numerical Analysis.** It should be noted that numerical investigation of the above system of equations is rather labor-consuming. An attempt to simplify this system was undertaken in [3]. The particle temperature was considered uniform, and the heat-influx equation for the particle was therefore converted to the ordinary differential equation

$$\frac{4}{3}\,\pi d^3 \rho_d c_d\,\frac{dT_d}{dt} = -4\pi d^2 q, \tag{2}$$

where $q$ is the heat flux through unit particle surface into the vapor:

$$q = \alpha_d (T_d - T_{gs}) \tag{3}$$

($\alpha_d$ is the coefficient of heat transfer from the particle). For $\alpha_d = \lambda_g/(\pi D_g t)^{1/2}$ and $\lambda_g = \mathrm{const}$, the particle temperature is described by the formula

$$T_d(t) = T_s + (T_d(0) - T_s)\exp\left(-(D_d t/a^2)^{1/2}\right). \tag{4}$$

Here $D = \lambda/(\rho C)$ is the thermal diffusivity.

The above formula was given without derivation but integrating Eq. (2) subject to relation (3) and the expression for $\alpha_d$, we obtain a different formula:

$$T_d(t) = T_s + (T_d(0) - T_s)\exp\Big[-6\,\frac{\lambda_g}{\lambda_d}\Big(\frac{D_d}{D_g \pi}\,\frac{D_d t}{d^2}\Big)^{1/2}\Big]. \tag{5}$$

An analysis of the above formulas for the particle temperature and a comparison with the numerical solution of the complete system of equations show that expression (5) adequately describes the particle temperature variation, whereas formula (4) overestimates the particle cooling rate.

Another simplifying assumption was that the vapor temperature profile was considered linear. Thus, the equation of heat transfer in the vapor was not solved, i.e.,

$$T_g = \frac{T_d - T_s}{d-a}\,(r-a) + T_s, \qquad \frac{\partial T_g}{\partial r} = \frac{T_d - T_s}{d-a}.$$

Because of this assumption, the pressure variation equation can be written as

$$\frac{dp_g}{dt} = \frac{3(\gamma-1)}{a^3-d^3}\Big((d^2-a^2)q_g - \frac{\gamma p_g}{\gamma-1}\,a^2 w_{ga}\Big), \tag{6}$$

where $q_g = -\lambda_g(\partial T_g/\partial r)\Big|_{\substack{r=a\\ r=d}} = \lambda_g(T_d - T_s)/(a-d)$ ($\lambda_g = \mathrm{const}$). This assumption leads to the contradiction that the amount of heat transferred to the vapor from the hot particle is smaller than the amount of heat transferred from the vapor to the cold liquid:

$$4\pi d^2 q_g < 4\pi a^2 q_g. \tag{7}$$

Finally, the last simplification is the specification of the heat flux from the vapor to the surrounding liquid. In [3], the following expression for the heat flux from vapor to the liquid was proposed:

$$q_l = a_l(T_s - T_\infty)a^2(t)/a^2(0). \tag{8}$$

Here $a_l = \lambda_l/(\pi D_l t)^{1/2}$.

The pressure dependence of the liquid saturation temperature was specified using an approximation of the Clausius–Clapeyron equation in the form

$$p_g = p_{g,0} \exp\Big(\frac{L}{R_g T_{s,0}}\Big(1 - \frac{T_{s,0}}{T_s}\Big)\Big). \tag{9}$$

Thus, the model proposed in [3] contains Eqs. (4), (6), (8), and (9) and the equations

$$p_g = \rho_g^0 R_g T_g, \qquad jL = q_g - q_l, \qquad j = \rho_l^0(a' - w_{la}) = \rho_g^0(a' - w_{ga}),$$

$$a' = \frac{da}{dt}, \qquad a\,\frac{dw_{la}}{dt} + \frac{3}{2}\,w_{la}^2 = \frac{p_g - p_\infty}{\rho_l^0}.$$

The calculation results confirm the validity of the assumption of uniform particle temperature. A comparison of the results of calculations using the model of [3] and the given model is given below. The results were obtained for a copper particle of radius $d = 0.2$ mm at a temperature $T_0 = 1356$ K in water ($p_0 = 10^5$ Pa). The initial radius of the vapor bubble $\bar{a}_0 = \bar{a}_0/d$ was set equal to 1.1, and the initial vapor temperature profile was considered linear.

Figure 1 shows a curve of particle temperature versus time plotted using formulas (4) and (5) ($t_* = d^2/D_g = 0.53$ sec). The particle temperature calculated by formula (5) changes little in the time interval considered.

Figure 2 gives time dependences of vapor pressure and bubble radius and Fig. 3 gives time dependences of the heat fluxes from the particle into the vapor, from the vapor to the bubble surface, and from the bubble surface into the liquid in the absence of a temperature drop in the liquid ($T_\infty = T_s$) at $t = 0$. The results of calculations using the given model (under the assumption that $\lambda_g = \text{const}$) are shown by a solid curve, and results of calculations using the model [3] are shown by a dashed curve [instead of formula (4) for $T_d$, formula (5) was used]. As noted above, the specification of a linear temperature profile leads to a change in the heat transfer process (Fig. 3). It should be noted that Fig. 3 gives values of the total heat fluxes from the entire surface accurate to $4\pi$, i.e., $Q = -R^2\lambda\,\partial T/\partial r$ and $Q_0 = 4\pi d\lambda_g T_0$.

From Fig. 2 it follows that the value of the maximum vapor pressure calculated for the model of [3] is underestimated. This is due to the negative contribution of the terms containing heat fluxes to the right side of Eq. (6). In addition, there is a significant difference between the growth rates of the vapor-layer thickness predicted by the models compared.

Results of calculation of the bubble radius for the case of underheated liquid are presented in Fig. 4. It is evident that according to the model considered, the bubble grows, whereas according to the model of [3], it collapses. This is due to the above-mentioned contradiction of the model of [3] [see inequality (7)].

It should be noted that in [3], the temperature dependence of the heat conductivity of vapor was ignored. In the present calculations, a model with a linear temperature profile taking into account the dependence $\lambda_g(T)$ was used. In this case, we have

$$\frac{dp_g}{dt} = \frac{3(\gamma - 1)}{a^3 - d^3}\Big(d^2 q_{gd} - a^2 q_{ga} - \frac{\gamma p_g}{\gamma - 1}\,a^2 w_{ga}\Big),$$

where $q_{gd} = -\lambda_g(T_d)(T_d - T_s)/(a - d)$ and $q_{ga} = -\lambda_g(T_s)(T_d - T_s)/(a - d)$.

Figures 5 and 6 give the same dependences as in Figs. 2 and 3 but obtained using the model with a linear temperature profiles taking into account the dependences $\lambda_g(T)$ and using the complete model with the variable $\lambda_g$. In Fig. 6a, it is evident that due to the variability of $\lambda_g$, the following condition is satisfied in the initial time interval:

$$4\pi d^2 q_{gd} > 4\pi a^2 q_{ga}.$$

As a result, in the initial time interval, the modified model of [3] with a linear temperature profile taking into account the variability of $\lambda_g$ is in better qualitative agreement with the solution of the total system of equations than the model of [3] (see Figs. 5 and 6b).

From a comparison of Fig. 5 and Fig. 2, it follows that accounting for the temperature dependence of the heat conductivity of the vapor leads to the activation of the process; therefore, neglect of this dependence can have a significant effect on the calculation results.

Thus, it is necessary to refine the assumptions adopted in [3], except for the assumption of uniform particle temperature, and to take into account the variability of the heat conductivity of the vapor.

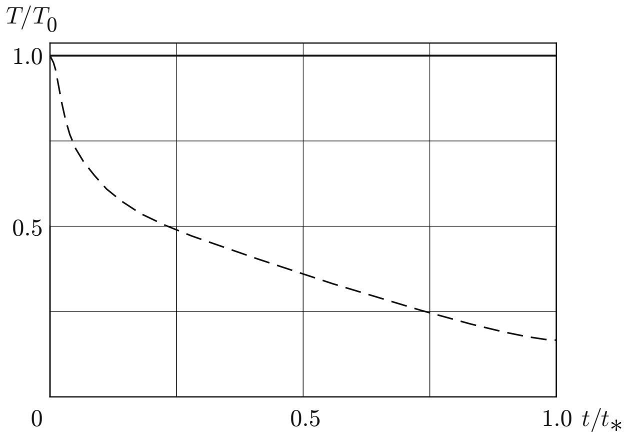


Fig. 1

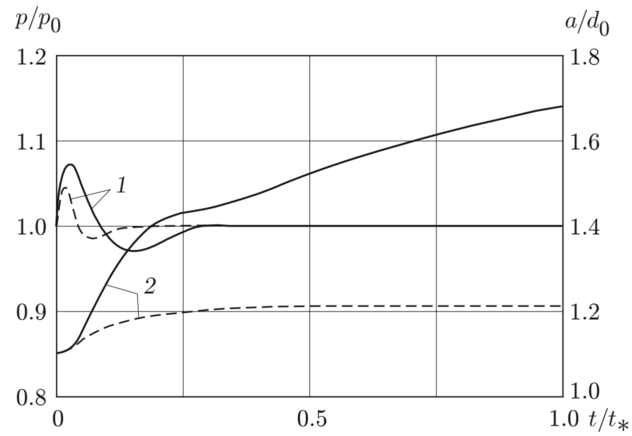


Fig. 2

Fig. 1. Particle temperature versus time ($T_0 = 1356$ K and $t_* = 0.53$ msec): the solid and dashed curves were calculated using formulas (5) and (4), respectively.

Fig. 2. Calculated vapor pressure (1) and bubble radius (2) ($p_0 = 10^5$ Pa, $d = 0.2$ mm, $T_0 = 1356$ K, $T_\infty = 373$ K, $a_0/d = 1.1$, and $t_* = 0.53$ msec): solid curves refer to the model considered ($\lambda_g =$ const); dashed curves refer to the model of [3].

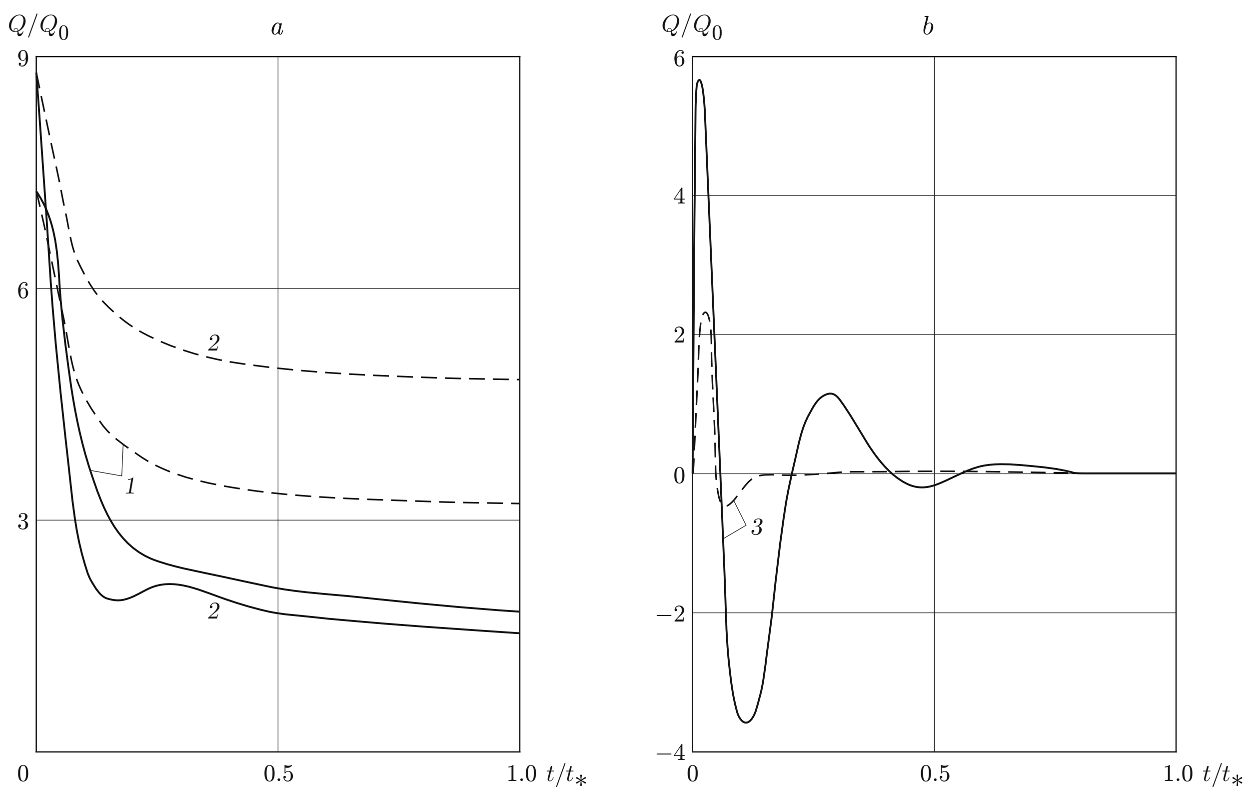


Fig. 3. Calculated heat fluxes ($p_0 = 10^5$ Pa, $d = 0.2$ mm, $T_0 = 1356$ K, $T_\infty = T_{s,0} = 373$ K, $a_0/d = 1.1$, and $t_* = 0.53$ msec): solid curves refer to the model considered ($\lambda_g =$ const); dashed curves refer to the model of [3]; 1) heat flux from the particle to the vapor; 2) heat flux from the vapor to the bubble surface; 3) heat flux from the bubble surface to the liquid.

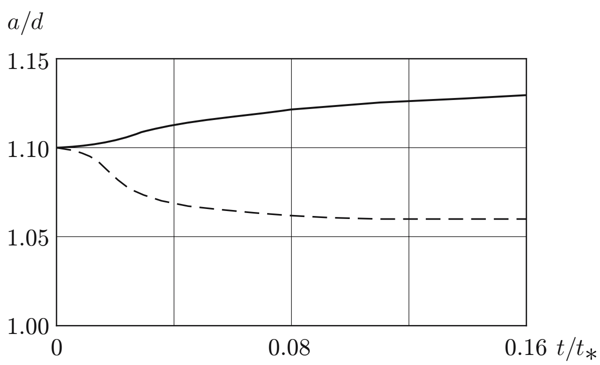


Fig. 4

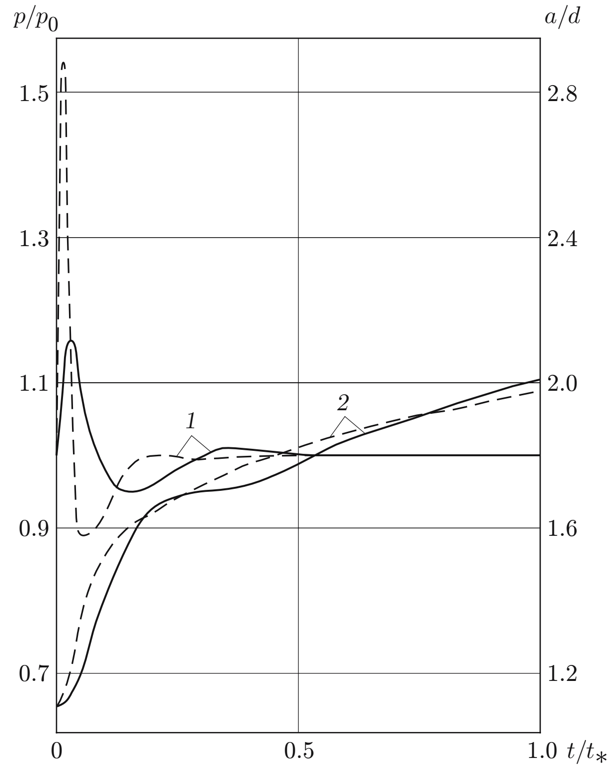


Fig. 5

Fig. 4. Calculated bubble radius for the case of underheated liquid ($p_0 = 10^5$ Pa, $d = 0.2$ mm, $T_0 = 1356$ K, $T_\infty = 363$ K, $T_{s,0} = 373$ K, $a_0/d = 1.1$, $t_* = 0.53$ msec, and $\lambda_g =$ const): the solid curve refers to the model considered; the dashed curve to the model of [3].

Fig. 5. Calculated vapor pressure (1) and bubble radius (2) ($p_0 = 10^5$ Pa, $d = 0.2$ mm, $T_0 = 1356$ K, $T_\infty = T_{s,0} = 373$ K, $a_0/d = 1.1$, $t_* = 0.53$ msec, and $\lambda_g \neq$ const): the solid curves refer to the model considered; the dashed curves to the model of [3].

A fairly complete and exact model of the process considered is proposed in [4], where the temperature fields in the vapor and surrounding liquid were determined form partial heat-influx equations and the particle temperature was determined from the ordinary differential equation derived under the assumption of uniform particle temperature. In this case, however, the temperature dependence of the heat conductivity of the vapor was ignored. If we compare the heat fluxes in the vapor phase on the bubble surface $q_{ga}$ and $q_{ga}^{\lambda}$ found from quasistationary temperature distributions (see below) for a constant and variable values of quantity $\lambda_g$, respectively, at $T_d = 1356$ K and $T_s = 373$ K, we obtain $g_{ga}^{\lambda}/g_{ga} \simeq 2.5$. The initial conditions in [4] were obtained by solving the problem of quasistationary temperature distributions in the vapor and liquid:

$$d \leqslant r \leqslant a_0: \qquad T_g = (T_{d,0} - T_{s,0})d(a_0/r - 1)/(a_0 - d) + T_{s,0}, \qquad w_{g,0} = 0,$$

$$a_0 \leqslant r: \qquad T_l = (T_{s,0} - T_\infty)a_0/r + T_\infty, \qquad w_{l,0} = 0.$$

On the bubble surface, the condition of equal heat fluxes in the liquid and vapor at the initial time was imposed:

$$\lambda_g \frac{\partial T_g}{\partial r}\Big|_{r=a} = \lambda_l \frac{\partial T_l}{\partial r}\Big|_{r=a},$$

whence it follows that

$$\frac{T_{d,0} - T_{s,0}}{T_{s,0} - T_\infty} = \frac{\lambda_l}{\lambda_g} \frac{a_0 - d}{d}.$$

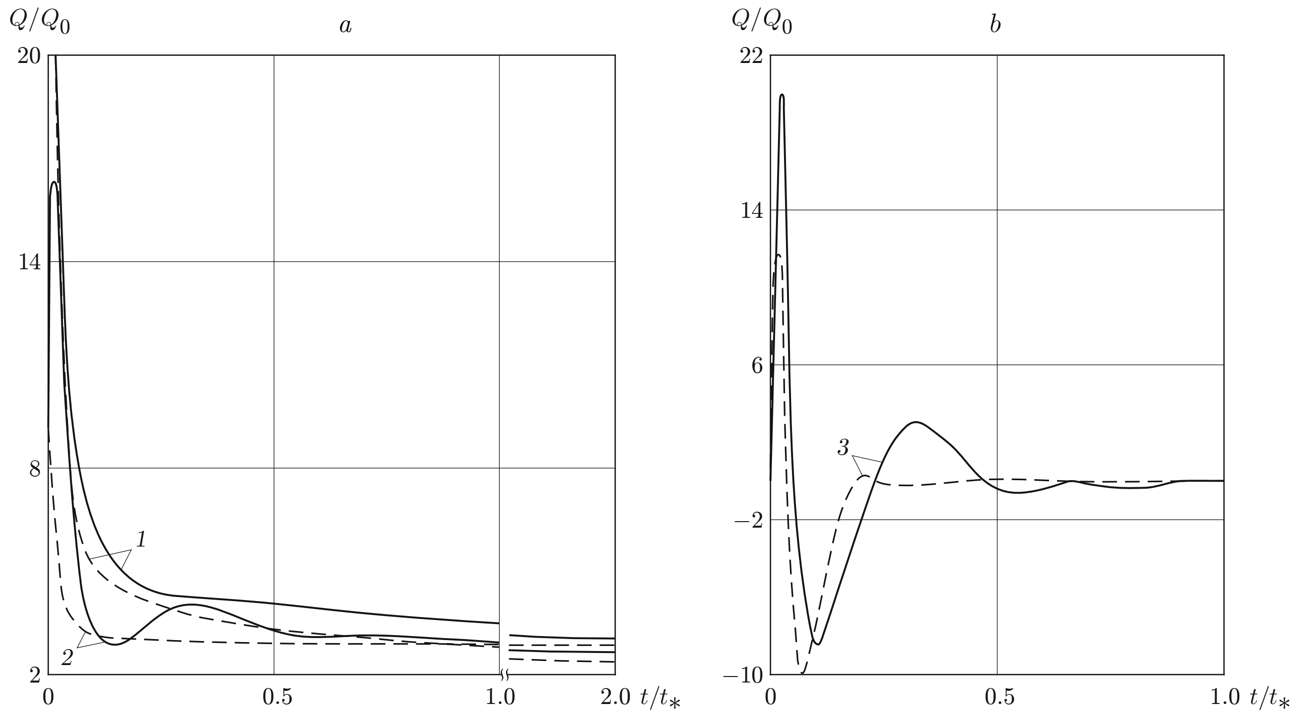


Fig. 6. Calculated heat fluxes ($p_0 = 10^5$ Pa, $d = 0.2$ mm, $T_0 = 1356$ K, $T_\infty = T_{s,0} = 373$ K, $a_0/d = 1.1$, $t_* = 0.53$ msec, and $\lambda_g \neq \text{const}$): solid curves refer to the model considered; dashed curves to the model with a linear temperature profiles; 1) heat flux from the particle to the vapor; 2) heat flux from the vapor to the bubble surface; 3) heat flux from the bubble surface to the liquid.

This relation significantly limits the range of initial conditions. For example, if the liquid temperature is equal to the boiling temperature, i.e., $T_\infty = T_{s,0}$, then for a finite value of $a_0$, the equality $T_{d,0} = T_{s,0}$ is satisfied.

Some results of numerical solution of the system of equations with an instantaneous increase or decrease in pressure at infinity at the initial time are presented in [4]. Unlike in [4], in the present study we consider another limiting case, namely instantaneous entry of particles into the liquid. In this case, the pressure away from the bubble does not vary and the motive power of the process is the initial temperature gradient, which can also cause radial oscillations of the bubble. Such formulations of the problem supplement each other.

An analysis of the complete model and the results of calculations using it shows that the time dependence of the growth rate of the vapor bubble is described by the equation

$$\bar{a}\,\frac{d^2\bar{a}}{d\tau^2} + \frac{3}{2}\Big(\frac{d\bar{a}}{d\tau}\Big)^2 = \frac{P_g - 2\sigma/(ap_\infty) - 1}{\varepsilon^2},$$

where $\varepsilon = d/[t_*(p_\infty/\rho_l^0)^{1/2}]$ is the parameter in the Rayleigh–Lamb equation, $t_* = d^2/D_g$, $P_g = p/p_\infty$, and $\tau = t/t_*$. According to this equation, the pressure deviation from the equilibrium value leads to a change in the growth rate of the vapor layer thickness. As the coefficient $\varepsilon$ increases (in particular, due to a reduction in the particle radius $d$), the system becomes less sensitive to pressure deviations from the equilibrium value and therefore admits more significant pressure amplitudes.

The process in question involves two stages: the initial dynamic stage and the subsequent thermal stage.

The initial conditions influence only the dynamic stage of the process $t < 0.1t_*$. This stage is characterized by pressure fluctuations and by a rapid growth of the vapor layer thickness. In this case, a reduction in the Peclet number $\text{Pe} = t_*/T$ ($T$ is the period of bubble pulsation) leads to increases in the pressure fluctuation amplitude and the duration of the dynamic stage.

In the thermal stage of the process, the behavior of the system is well described by the quasistationary solution of the heat-conduction equations. For vapor, it has the form

$$\lambda_g(T_g)\,\frac{\partial T_g}{\partial r} = \frac{C_1}{r^2}, \qquad \lambda_g(T) = AT_g + B,$$

$$AT_g^2/2 + BT_g = -C_1/r + C_2, \tag{10}$$

$$C_1 = -\frac{ad}{a-d}\,(T_d - T_s)\,\frac{\lambda_g(T_d) + \lambda_g(T_s)}{2}, \qquad C_2 = \frac{C_1}{a} + \frac{1}{2}\,AT_s^2 + BT_s.$$

The obtained quasistationary solution of the heat-conduction equation for vapor describes the behavior of the system in the thermal stage ($t > 0.1t_*$) with high accuracy, and qualitatively describes the behavior in the dynamic stage of the process.

A reduction in the initial thickness of the vapor layer leads to an increase in the duration of the dynamic stage and activation of the processes at this stage. An important feature of the problem is the temperature dependence of the heat conductivity of the vapor.

Assuming that the vapor particles inside the shell are motionless and that the shell radius increases only as a result of vaporization of the liquid, we have

$$j = \rho_g(a)a', \tag{11}$$

where $\rho_g(a)$ is the vapor density on the outer boundary of the vapor layer.

Let us consider the limiting case of the maximum growth of the vapor shell thickness where the particle does not cool and the entire heat transferred to the outer boundary of the vapor shell from the heated particle is expended in vaporizing the liquid, i.e., $q_{la} = -\lambda_l(\partial T_l/\partial r)_a = 0$. This condition corresponds to the case where highly heated particles enter the boiling liquid ($T_l = T_s$) or to the limiting case of a liquid with low heat conductivity ($\lambda_l \to 0$).

In view of (10) and (11), the boundary condition on the bubble surface (1) is written as

$$C_1/a^2 = \rho_{gs}\dot{a}L. \tag{12}$$

In the asymptotic stage of growth where $a \gg d$, the solution of Eq. (12) has the form

$$a = \sqrt[3]{\varkappa t}, \qquad \varkappa = \frac{3}{2}\,\frac{d(T_d - T_s)}{L\rho_{gs}}\,[\lambda_g(T_d) + \lambda_g(T_s)].$$

The above relation is an upper-bound estimate since it was obtained ignoring the fact that the particle cools as the vapor shell thickness increases and that part of the heat flux emitted by the particle is dissipated into the liquid. However, in this limiting case, too, the shell thickness increases much more slowly than in the asymptotic stages of vapor bubble growth in an overheated liquid, where the bubble radius increases in proportion to $t^{1/2}$ [10].

An important characteristic of a vapor explosion is the magnitude of the acoustic impulse due to film boiling on the particles. Let us assume that all hot particles have identical sizes and are uniformly distributed within the spherical zone of interaction. As the film vapor around each particle grows, the interaction zone increases and an acoustic impulse occurs. To determine the impulse, we use the following formula for the pressure on the surface of the interaction zone [3, 11]:

$$\frac{p}{p_0} = 1 + \frac{\rho_0 R^2 \beta \bar{a}}{p_0}\,(1 + \bar{a}^3\beta)\Big(2\Big(\frac{d\bar{a}}{dt}\Big)^2 + \bar{a}\,\frac{d^2\bar{a}}{dt^2}\Big). \tag{13}$$

Here $\bar{a} = a/a_0$, $\beta$ is the volume fraction of hot particles, and $\rho_0$ and $R$ are the density and radius of the interaction zone at the initial time, respectively. Thus, it is necessary to consider the problem of growth in the vapor shell thickness around a single particle.

Using the results of calculations for the complete model presented in Figs. 5 and 6 ($\beta = 0.001$ and $R = 0.1$), from (13) we find that the value of the impulse is approximately equal to $43 \cdot 10^5$ Pa.

Thus, the procedure described above can be used to estimate the maximum impulse that occurs during vapor explosion.

The complete system of basic equations for solving the problem considered is given in [11]. Later, some aspects of this issue were studied in [12–14].

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